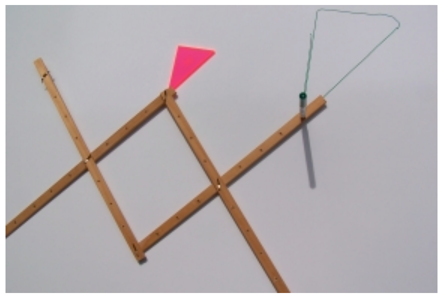
Print
Assign
Submit
Solution and Commentary
Online Resources
Teacher Packet [pdf]
Strategy 11: Get Unstuck
Strategy Examples [pdf]

Pondering Pantographs [Problem #5271]

Pantographs are mechanical devices that were used as far back as 1603 to produce copies (sometimes larger, sometimes smaller, sometimes the same size) of pictures or text.

Here is a picture of one that we made:



Click on any of the photos for a larger version

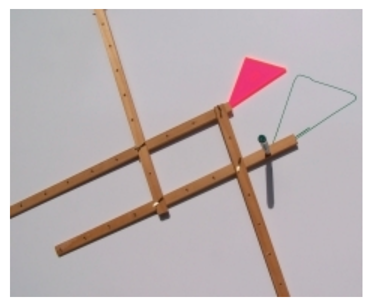
The four arms form a parallelogram. The end of one arm is fixed (the upper left in our picture). The end of another arm holds the pen (the green marker). And the vertex of the parallelogram between the fixed point and the pen traces the original object (pink triangle). Those three points (pen, tracer, and fixed point) are all on the same line. The angles of the parallelogram can change while you trace, but the lengths cannot.

We wondered how the math of the pantograph works. Why is the drawing exactly like the original shape, only a different size?

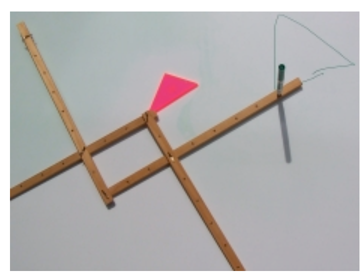
Extra: In our first picture above, the parallelogram was actually a rhombus. But it can be any parallelogram. Since we built ours out of wood, with holes drilled every two inches, it was easy to change it.

We traced the triangle again. (The fixed point is just off the screen, in the very next hole.)

Describe how to tell what the scale factor will be for any pantogram.



Extra Extra: We wondered if it was important that those three points of the pantograph were on the same line. On the right is a drawing we tried where the points aren't on the same line. What do you think?



Comments and Sample Solutions

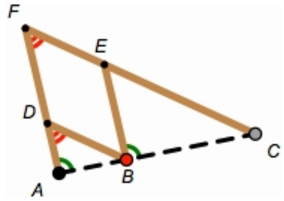
We received a number of good efforts for this problem, but none of them had all the math quite right. We've decided to share our solution. It does sound like some of you have used pantographs before, but never tried to figure out how they work. The inspiration for this problem came from an activity in a *Geometer's Sketchpad* book in which students construct a pantograph. The book is targeted toward middle school students, but I have used it with high school students and then given them the homework of showing why the whole contraption works.

In the picture below, A is the fixed point, B is the tracer, and C is the pen. We know that DFEB is a parallelogram and that A, B, and C are collinear.

Angles DAB and EBC are congruent since AF and BE are parallel (because it's a parallelogram), AC is a transversal across the parallel lines, and those angles are corresponding angles.

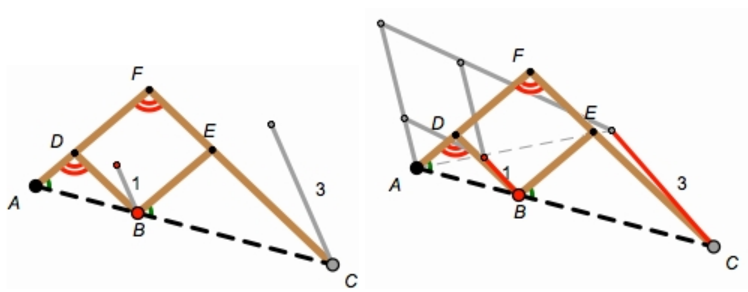
Angles ADB and AFC are congruent using the same reasoning (with AF the transversal across DB and FE).

Triangles ADB and AFC must be similar because of AA.



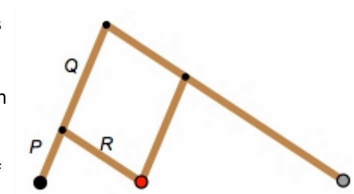
Because of the similar triangles and the fact that their corresponding sides are in proportion, the ratio of AD to AF is the same as the ratio of AB to AC. The ratio of AD to AF doesn't change, since AD and AF are made of wood and have fixed lengths. The angles change, but the lengths don't.

For example, say AD is 1 and AF is 3, then that ratio is 1 to 3. So the ratio of AB to AC is also 1 to 3. Every time B moves 1 inch, C will move 3 inches. It will move in the same direction since A is fixed.



The first above shows the new position of the pantograph after the move and the paths that B and C took. The second picture is the same thing, plus the original position of the pantograph in gray.

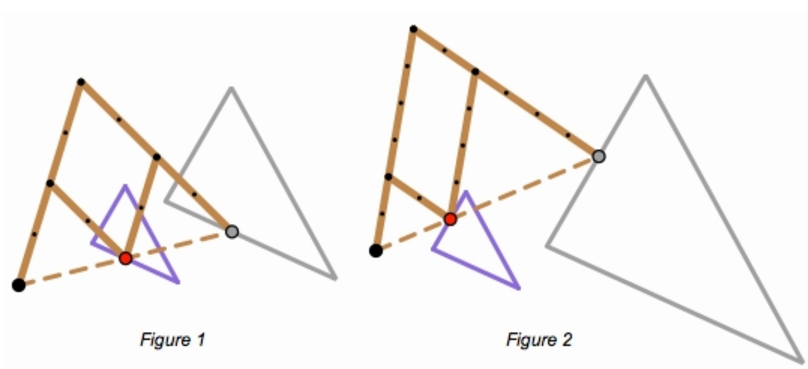
Extra: When making a pantograph, we can change the lengths of P, Q, and R. The rest follows because (1) there has to be a parallelogram, and (2) the pen has to be on the same line as the stationary point and the tracer.



In the picture in the main problem, it looks like all of those parts (P, Q, and R) are four units long. So I started with an example in which they're all equal (they are 2 units long in this case). This is shown in Figure 1 below.

Working with the ratios used in the first part of the problem, we can see that AD:AF is 1:2. So the ratio of the size of the original figure to the new figure is 1:2. The dimensions of the new triangle are twice those of the original triangle.

In the example shown in Figure 2 below, AD:AF is 1:3. So the dimensions of the new figure will be 3 times those of the original.



I'm realizing that turning the parallelogram into something that isn't a rhombus doesn't make any difference. All that matters is AD:AF. So in Figure 3 below, AD is 2 and AF is 6. So even though it's a parallelogram instead of a rhombus, the dimensions of the image will still be 3 times those of the original. In Figure 4, AD is 2 and AF is 3, so the dimensions of the image will be 1.5 times the original.

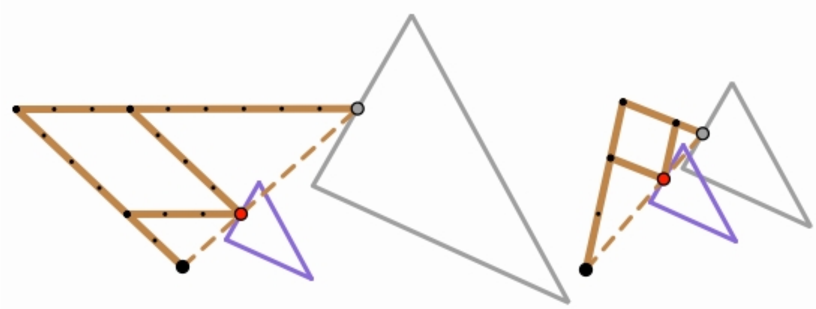


Figure 3 Figure 4

All you need to know to determine the scale factor of any pantograph is the ratio of AD to AF.

There are no solutions available for this problem.